Range Commanders Council

DATA REDUCTION AND
COMPUTER GROUP

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GLOBAL COORDINATE SYSTEM

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GLOBAL COORDINATE SYSTEM

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I. INTRODUCTION

The purpose of this document is to define and briefly discuss coordinate systems commonly employed at test ranges as well as to recommend procedures and practices for the use of a global coordinate system. This document replaces IRIG Document 151-69 (formerly 103-69), Global Coordinate System.

This document is primarily intended for use by individuals familiar with the basic concepts and definitions commonly employed in geodesy. In order to aid the reader who requires additional background information, a bibliography of some of the standard texts on geodesy, map projections, and spherical astronomy is provided.

This document specializes in those trajectory coordinate systems that are, or could be, potentially applicable to data reduction tasks performed at the various test ranges. No attempt has been made to address the subject of coordinate systems in a general manner or to develop a compendium of the various coordinate systems employed at the test ranges. General discussions on coordinate systems are given in a number of standard texts on geodesy and are not repeated in this document. Information on special-purpose coordinate systems employed at a given range can usually be obtained from documents at the local installation. A compendium of all coordinate systems used by member ranges would produce a voluminous document of dubious value. In spite of being narrow in scope, this document still manages to serve a useful purpose through an in-depth presentation of the definitions, properties and useful transformations associated with those coordinate systems having a general interest to data reduction specialists at most test ranges.

II. GLOBAL COORDINATES

Any global coordinate system (geodetic datum) is based on an ellipsoid of revolution whose size and shape approximate that of the entire Earth and whose center coincides in some manner with the gravitational center of the Earth. Examples of such systems are the Mercury Datum based on the Fischer 1960 Ellipsoid and the DOD World Geodetic System (WGS) 1972.

Satellite and missile trajectories are relative to the Earth's center of gravity; therefore, range data referred to the center of gravity are the end products for such trajectory analyses. When the sensor coverage exceeds the extent of local or regional datums, the sites must be located on a global datum to achieve consistent geometry.

Global coordinates based on a given ellipsoid are determined by transformations from regional or local coordinate systems or from observations of a satellite whose orbit is known relative to some coordinate system. These computations and transformations are usually assumed to lead to consistent results. However, when it is possible to make comparisons, the results are rarely in complete agreement. Differences in computational and orbital force field models, along with parameter and computer characteristics, are responsible for such discrepancies. Additionally, in theory, the centers of two global datum systems should only differ by a constant; however, in practice, the difference appears to be a function of the location of the measurements.

III. REGIONAL COORDINATES

A regional geodetic datum consists of an ellipsoid whose size and shape approximate that of the particular region of the Earth. This ellipsoid is oriented to a physical point in the region, and this point is the origin of the regional datum. Examples of such systems and their reference ellipsoids are the European Datum (ED) on the International Ellipsoid of 1924, the North American Datum (NAD) on the Clarke 1866 Ellipsoid, and the Tokyo Datum (TD) on Bessel 1841 Ellipsoid.

Conventional surveys in a region are referenced to the datum network defining the regional system. When distance and azimuth between regional sensors are necessary for scaling, such regional coordinates are the best computational source. Regional coordinates may be extended to greater distances by optical and electronic observation of a satellite. The relation between regional and global coordinate systems is specified by displacement between ellipsoid centers in rectangular coordinates (ΔE, ΔF, ΔG) and differences in ellipsoid sizes and shapes (Δa, Δf), where a and f are semi-major axis and flattening, respectively.

IV. PARTICULAR RANGE (LOCAL) COORDINATES

These coordinate systems are usually rectangular systems whose center (origin) and x-y plane are tangent to some convenient point on the range (usually one of the launch pads or instrumentation sites). The coordinates may be transformed from these local coordinates to appropriate regional coordinates or global systems. These local range systems are primarily a computational convenience designed for the range configuration and mission. Such systems are not amenable to interrange projects or missions.

V. DATUM TRANSFORMATIONS

Frequently, it is necessary to transform station geodetic coordinates from one datum to another. The following information is necessary for this transformation:

a_0 = the semi-major axis of the reference ellipsoid of the datum from which the transformation will occur (original datum),

f_0 = the flattening of the reference ellipsoid of the original datum,

a_N = the semi-major axis of the reference ellipsoid for the datum to which the transformation will occur (new datum),

f_N = the flattening of the reference ellipsoid of the new datum, and

$$\left.\begin{aligned} \Delta E &= E_N - E_0 \\ \Delta F &= F_N - F_0 \\ \Delta G &= G_N - G_0 \end{aligned}\right\}$$ the displacement of the center of the new datum relative to the original

The EFG system is a right-handed Cartesian system. The origin of this system is at the geometric center of the reference ellipsoid. The coordinate axes are oriented as follows: E and F lie in the plane of the Equator, G coincides with the rotational axis of the Earth and is positive through the North Pole, E is positive through Greenwich Meridian (0° longitude), and F is positive to complete a right-handed system. This system rotates with the Earth and is Earth-centered and Earth-fixed (ECEF). The fact that the various Earth-centered systems exhibit displacement of their centers is evidence of experimental error rather than disagreement in fundamental assumptions. This EFG system will be called the Master Coordinate System or the system in which integration takes place.

The method of transformation consists of the following steps:

1. Calculate the geocentric Cartesian coordinates of the original coordinates given ϕ_0, λ_0, H_0, a_0, f_0, where ϕ_0, λ_0 and H_0 are the geodetic latitude, longitude and height above the reference ellipsoid, respectively, of the point to be transformed. The method of transformation, discussed in section VII, paragraph 7, produces the data set (E_0, F_0, G_0).

2. Calculate the geocentric Cartesian coordinates of the new datum:

$$E_N = E_0 + \Delta E$$

$$F_N = F_0 + \Delta F$$

$$G_N = G_0 + \Delta G$$

3. Calculate ϕ_N, λ_N, and H_N using a_N, f_N, E_N, F_N, and G_N as shown in section VII, paragraph 7.

The origins and ellipsoids for a few datums are

	NAD 1927 Meade's Ranch, KS	ED Potsdam	TD Tokyo
ϕ	39°13'26.686"N	52°22'51.45"N	35°39'17.51"N
λ	261°27'29.494"E	13°03'58.93"E	139°44'40.90"E
H	599.4M±0.3		
Ellipsoid	Clarke 1866	International 1924	Bessel 1841

The ellipsoid parameters of interest are

Ellipsoid	a (meters)	f
Clarke 1866	6378206.4	1/294.9786982
Fischer (Mercury) 1960	6378166	1/298.3
Kaula 1961	6378165	1/298.3
Bessel 1841	6377397.155	1/299.1528128
WGS 72	6378135	1/298.26
International (Hayford) 1924	6378388	1/297.0
WGS 84	6378137	1/298.257223563

VI. TRAJECTORY COORDINATE SYSTEMS

Orbital computations involve a number of different coordinate systems and transformation of data among these systems. These coordinate systems are related to each other through a reference figure representing the actual Earth. This reference figure is an ellipsoid of revolution of given dimensions whose surface is assumed to approximate closely the mean sea level surface of the actual Earth. It is also assumed that the mass of the Earth is sufficiently homogeneous for its center to be taken at the geometric center of the reference ellipsoid. In this way, the major axis of the reference ellipsoid lies in the equatorial plane of the Earth, and the minor axis coincides with the Earth's rotational axis.

The location and origin of a given data acquisition coordinate system are determined by an astronomical and a geodetic survey (the latter of which is based upon a specific ellipsoid). The geodetic coordinates are latitude ϕ, longitude λ, and height H above the surface of the reference ellipsoid. The geodetic latitude of a point, which is positive in the Northern Hemisphere, is the angle between the equatorial plane and the geodetic vertical through the point extended to intersect the plane of the equator. The geodetic longitude of a point is an angle in the equatorial plane with its vertex at the center of the Earth, its initial side through the Greenwich Meridian, and its terminal side through the meridian of the point. Longitude is measured positive eastward of Greenwich. Geodetic height is measured positive up from the surface of the reference ellipsoid along the geodetic vertical.

The astronomical coordinates are latitude ϕ' and longitude λ'. The astronomical latitude is defined as the angle between the equatorial plane and the astronomical vertical through the point. The astronomical vertical is defined by the direction of gravity (plumbline) at the point. The astronomical longitude is the angle between the plane containing the Greenwich Meridian and the projection of the astronomical vertical on the equatorial plane. The latitude and longitude differences between geodetic and astronomical coordinates define the deflection of the vertical. Astronomical coordinates define only orientation, while geodetic coordinates define orientation and location. This distinction is of importance to the transformation equations detailed in the next section.

Prior to any orbital computation, a set of initial conditions must be obtained if equations of motion are to be integrated. Position and velocity are usually needed (and ballistic coefficient and covariance matrix, if possible). The initial conditions may be transformed to the ECEF system.

VII. PARTICULAR TRAJECTORY COORDINATE SYSTEMS

1. Topocentric Range Coordinate System $(x, y, z, \dot{x}, \dot{y}, \dot{z})$

A topocentric Cartesian system is assumed with the x-y plane normal to the geodetic normal determined by the origin, with geodetic coordinates ϕ, λ and H. The xyz range system is shown in figure 1-1. The coordinate system is right-handed with positive x pointing east, positive y pointing north, and positive z pointing up along the normal completing the right-handed system.

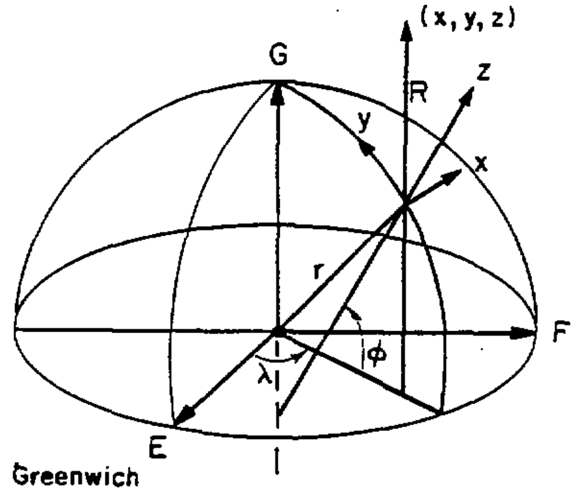


Figure 1-1

If the slant range azimuth, and elevation are known, the position vector components are

$$x = R \cos \varepsilon \sin \alpha$$

$$y = R \cos \varepsilon \cos \alpha \qquad (1)$$

$$z = R \sin \varepsilon$$

where

R is the slant range from the origin to a point with topocentric coordinates (x,y,z),

ε is the elevation angle of the position vector above the x-y plane, and

α is the azimuth angle measured in the x-y topocentric plane from True North to the projection of the position vector on the x-y plane.

If the components of the position vector x, y, z are known, then R, α, ε may be determined from the following equations:

$$R = (x^2 + y^2 + z^2)^{\frac{1}{2}} \tag{2}$$

$$\alpha = \tan^{-1}\left(\frac{x}{y}\right) \quad \text{(Double Arctan)} \quad (\text{If } = \alpha < 0, \text{ then } \alpha = \alpha + 2\pi) \tag{3}$$

$$\varepsilon = \tan^{-1}\left(\frac{z}{(x^2 + y^2)^{\frac{1}{2}}}\right) \tag{4}$$

In general, position and range data are the basic measurements received by instrumentation systems that are external to a vehicle in flight. Most methods of obtaining velocity and acceleration make use of numerically differentiated filtered position data (expressed as a function of time). If the position and velocity components are known, the range rate, azimuth rate, and elevation rate may be computed as follows:

$$\dot{R} = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{R} \tag{5}$$

$$\dot{\alpha} = \frac{y\dot{x} - x\dot{y}}{x^2 + y^2} \quad \text{(radians per second)} \tag{6}$$

$$\dot{\varepsilon} = \frac{\dot{z} - z(\dot{R}/R)}{(x^2 + y^2)^{\frac{1}{2}}} \quad \text{(radians per second)} \tag{7}$$

2. Geocentric Coordinate System (Master) $(E, F, G, \dot{E}, \dot{F}, \dot{G})$

We assume a right-handed Cartesian system located at the center of the ellipsoid of reference with the E-F plane fixed in the equatorial plane (that is ECEF) and the G axis along the rotational axis of the Earth. E is positive in the direction of longitude (λ)= 0° and latitude (ϕ) = 0°. F is positive 90° counterclockwise from E, and G is positive in the direction ϕ = 90° (see figures 2-1 and 2-2).

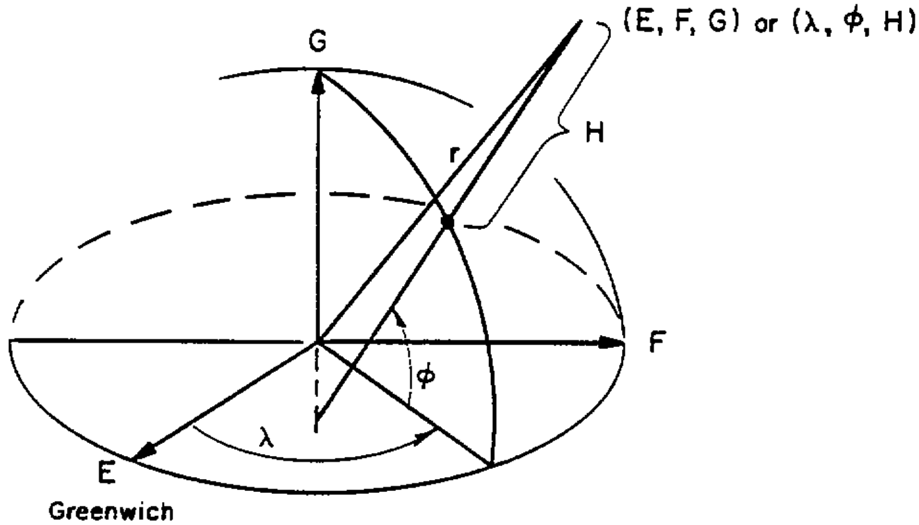


Figure 2-1

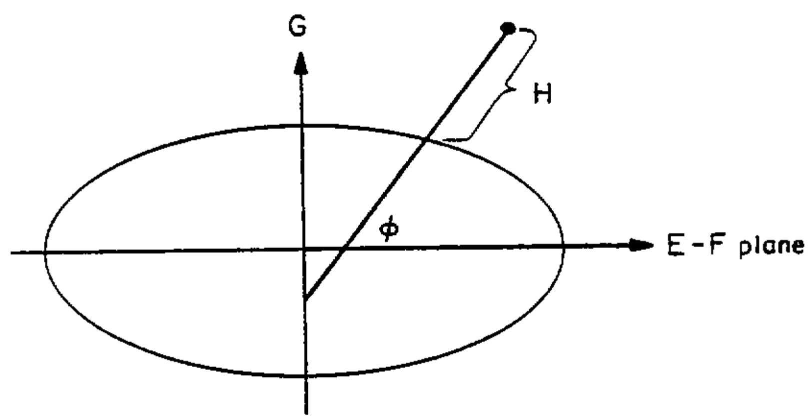


Figure 2-2

Prior to any transformation of topocentric coordinates, corrections for deflection from the vertical must be applied. Assuming these corrections have been made, the transformation of position and velocity components measured in the topocentric range system is given by

$$\begin{vmatrix} E \\ F \\ G \end{vmatrix} = \begin{vmatrix} -\sin\lambda_o & -\cos\lambda_o & 0 \\ \cos\lambda_o & -\sin\lambda_o & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \sin\phi_o & -\cos\phi_o \\ 0 & \cos\phi_o & \sin\phi_o \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} + \begin{vmatrix} E_o \\ F_o \\ G_o \end{vmatrix} \quad (8)$$

or

$$\begin{vmatrix} E \\ F \\ G \end{vmatrix} = \begin{vmatrix} -\sin\lambda_o & -\cos\lambda_o \sin\phi_o & \cos\lambda_o \cos\phi_o \\ \cos\lambda_o & -\sin\lambda_o \sin\phi_o & \sin\lambda_o \cos\phi_o \\ 0 & \cos\phi_o & \sin\phi_o \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} + \begin{vmatrix} E_o \\ F_o \\ G_o \end{vmatrix} \quad (9)$$

The geocentric coordinates of the topocentric reference site are computed from

$$\begin{aligned} E_o &= (N_o+H_o)\cos\phi_o \cos\lambda_o \\ F_o &= (N_o+H_o)\cos\phi_o \sin\lambda_o \\ G_o &= [N_o(1-e^2) + H_o] \sin\phi_o \end{aligned} \quad (10)$$

The parameter N_o in equation (10) is the transverse radius of curvature given by

$$N_o = \frac{a_o}{(1-e^2\sin^2\phi_o)^{\frac{1}{2}}} \quad (11)$$

where

a_o is the semi-major axis of the reference ellipsoid,

e is the eccentricity of the reference ellipsoid,

ϕ_o is the geodetic latitude of the reference site, positive north of the Equator, and

λ_o is the geodetic longitude of the reference site, positive east.

In addition, H_o is the geodetic height of the reference site and

$$H_o \cong n + h$$

where n is the geoid-ellipsoid separation and h is the height of the reference site above (or below) mean sea level.

The geocentric velocities are found from the following transformations:

$$\begin{vmatrix} \dot{E} \\ \dot{F} \\ \dot{G} \end{vmatrix} = \begin{vmatrix} -\sin \lambda_o & -\cos \lambda_o \sin \phi_o & \cos \lambda_o \cos \phi_o \\ \cos \lambda_o & -\sin \lambda_o \sin \phi_o & \sin \lambda_o \cos \phi_o \\ 0 & \cos \phi_o & \sin \phi_o \end{vmatrix} \begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{vmatrix} \qquad (12)$$

or

$$\begin{vmatrix} \dot{E} \\ \dot{F} \\ \dot{G} \end{vmatrix} = K \begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{vmatrix} \qquad (13)$$

where K is the matrix in equation (12).

If the geocentric coordinates of the point are known, the topocentric coordinates may be found by

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = K^T \begin{vmatrix} E - E_o \\ F - F_o \\ G - G_o \end{vmatrix} \qquad (14)$$

If the topocentric coordinates are positioned so that the x axis is at some angle from north other than 90°, the K matrix becomes

$$K = \begin{vmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{vmatrix} \tag{16}$$

where

K_{11} = -sin λ sin α - cos α cos λ sin ϕ,

K_{12} = sin λ cos α - sin α cos λ sin ϕ,

K_{13} = cos λ cos ϕ,

K_{21} = cos λ sin α - cos α sin λ sin ϕ,

K_{22} = -cos λ cos α - sin α sin α sin ϕ,

K_{23} = sin λ cos ϕ,

K_{31} = cos ϕ cos α,

K_{32} = cos ϕ sin α, and

K_{33} = sin ϕ,

where α is the angle from north to the position of the positive x axis in the the topocentric system (see figure 2-3).

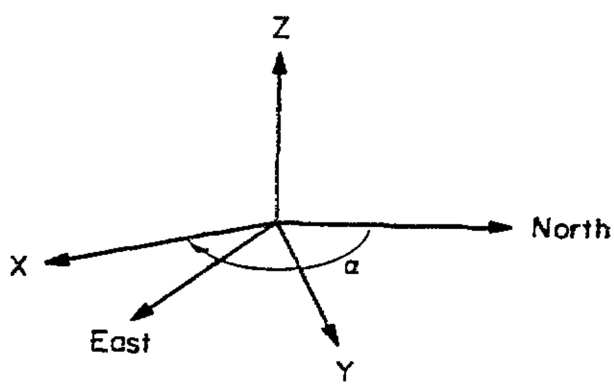


Figure 2-3

3. Earth-Fixed Spherical Coordinate System (r, λ, δ, v_e, A_e, γ_e)

A spherical system is assumed with the same origin as the ECEF (EFG) coordinate system (see figure 3-1). In addition, a UVW coordinate system is also introduced. The latter is a right-handed coordinate system with U pointing east, V pointing north, and W pointing along the projection of the geocentric radius r. The U-V plane is normal to the geocentric radius (see figure 3-1).

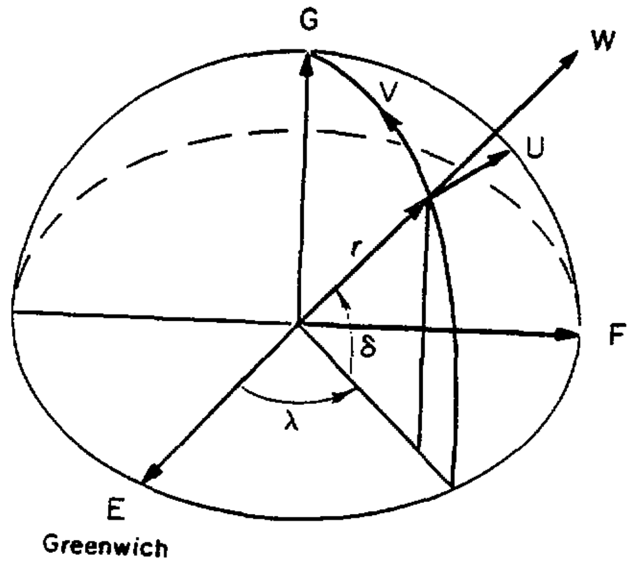


Figure 3-1

The following parameters can now be defined in terms of these coordinate systems:

r is the geocentric range from the center of the reference spheroid to the point (origin of the (UVW) coordinate system).

δ is the declination, positive north. The declination is the angle between the equatorial plane and the geocentric range vector.

λ is the longitude, positive east.

v_e is the Earth-fixed total velocity.

A_e is the azimuth of the velocity vector projected on the U-V plane. The angle is measured from north to the projection of the velocity vector. A_e is commonly called the heading.

γ_e is the elevation of the velocity vector above the U-V plane. γ_e is commonly called the flight path angle.

If the position vector components of a point in space are given in the ECEF system, then r, λ and δ may be found from

$$r = (E^2 + F^2 + G^2)^{\frac{1}{2}} \tag{17}$$

where E, F, and G are the geocentric coordinates of the point,

$$\delta = \tan^{-1}\left(\frac{G}{(E^2 + F^2)^{\frac{1}{2}}}\right), \tag{18}$$

and

$$\lambda = \tan^{-1}\left(\frac{F}{E}\right). \quad \text{(Double Arctan)} \; (\text{If } \lambda<0, \text{ then } \lambda = \lambda + 2\pi) \tag{19}$$

If the coordinates of the point in space are given in polar coordinates r, λ, δ, the geocentric coordinates are

$$E = r \cos \delta \cos \lambda$$

$$F = r \cos \delta \sin \lambda \tag{20}$$

$$G = r \sin \delta$$

If the geocentric velocities are known, they can be transformed from the EFG coordinate system to the UVW coordinate system with the following rotations:

$\theta_1 = \lambda$

$\theta_2 = 90°$

$\theta_3 = (90° - \delta)$

The transformation is

$$\begin{vmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{vmatrix} = R_3 R_2 R_1 \begin{vmatrix} \dot{E} \\ \dot{F} \\ \dot{G} \end{vmatrix} \tag{21}$$

where

$$R_3R_2R_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & \sin\theta_3 \\ 0 & -\sin\theta_3 & \cos\theta_3 \end{vmatrix} \begin{vmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{22}$$

Substituting the values of θ_1, θ_2, and θ_3 into the $R_3R_2R_1$ matrix and matrix multiplying gives

$$R_3R_2R_1 = \begin{vmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\delta\cos\lambda & -\sin\delta\sin\lambda & \cos\delta \\ \cos\delta\cos\lambda & \cos\delta\sin\lambda & \sin\delta \end{vmatrix} \tag{23}$$

The result of the transformation using equations (21) and (23) is

$$\begin{aligned} \dot{U} &= -\dot{E}\sin\lambda + \dot{F}\cos\lambda \\ \dot{V} &= -\dot{E}\sin\delta\cos\lambda - \dot{F}\sin\delta\sin\lambda + \dot{G}\cos\delta \\ \dot{W} &= \dot{E}\cos\delta\cos\lambda + \dot{F}\cos\delta\sin\lambda + \dot{G}\sin\delta \end{aligned} \tag{24}$$

The transpose of the $R_3R_2R_1$ matrix is

$$(R_3R_2R_1)^T = \begin{vmatrix} -\sin\lambda & -\sin\delta\cos\lambda & \cos\delta\cos\lambda \\ \cos\lambda & -\sin\delta\sin\lambda & \cos\delta\sin\lambda \\ 0 & \cos\delta & \sin\delta \end{vmatrix} \tag{25}$$

and

$$\begin{vmatrix} \dot{E} \\ \dot{F} \\ \dot{G} \end{vmatrix} = (R_3R_2R_1)^T \begin{vmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{vmatrix} \tag{26}$$

If the geocentric velocities ($\dot{E}$, $\dot{F}$, $\dot{G}$) of the point in space are known, v_e, A_e, λ_e may be found from the velocity components transformed to the UVW coordinate system by the use of equation (24) and the following relations:

$$v_e = (\dot{U}^2 + \dot{V}^2 + \dot{W}^2)^{\frac{1}{2}} \qquad (27)$$

$$A_e = \tan^{-1}\left(\frac{\dot{U}}{\dot{V}}\right) \qquad \begin{matrix}\text{(Double Arctan)} \\ (\text{If } A_e < 0, \text{ then } A_e = A_e + 2\pi)\end{matrix} \qquad (28)$$

$$\gamma_e = \tan^{-1}\left(\frac{\dot{W}}{(\dot{U}^2 + \dot{V}^2)^{\frac{1}{2}}}\right) \qquad (29)$$

If, on the other hand, the total velocity, heading, and flight path angle are known in the UVW coordinate system, then

$$\begin{aligned} \dot{U} &= v_e \cos\gamma_e \sin A_e \qquad (30) \\ \dot{V} &= v_e \cos\gamma_e \cos A_e \\ \dot{W} &= v_e \sin\gamma_e \end{aligned}$$

The geometric velocities may be obtained in this case by transforming the $\dot{U}$, $\dot{V}$, $\dot{W}$ to the $\dot{E}$, $\dot{F}$, $\dot{G}$ (ECEF) system with equation (26).

4. Cartesian Inertial Coordinate System (E_I, F_I, G_I, $\dot{E}_I$, $\dot{F}_I$, $\dot{G}_I$)

In figure 4-1, an Earth-centered inertial (ECI) coordinate system is represented by E_I, F_I, G_I. The ECEF rotating coordinate system is represented by E, F, G. The ECEF system is rotating about the G_I(G) axis with an angular velocity $\dot{\theta}$. ($\dot{\theta} = \omega_e$, the rotational rate of the Earth.) By definition, E_I is positive toward the mean vernal equinox of date, F_I is positive 90° counterclockwise from E_I, and G_I completes the right-handed system. E_I and F_I are in the Earth's equatorial plane, and G_I is positive toward the North Pole.

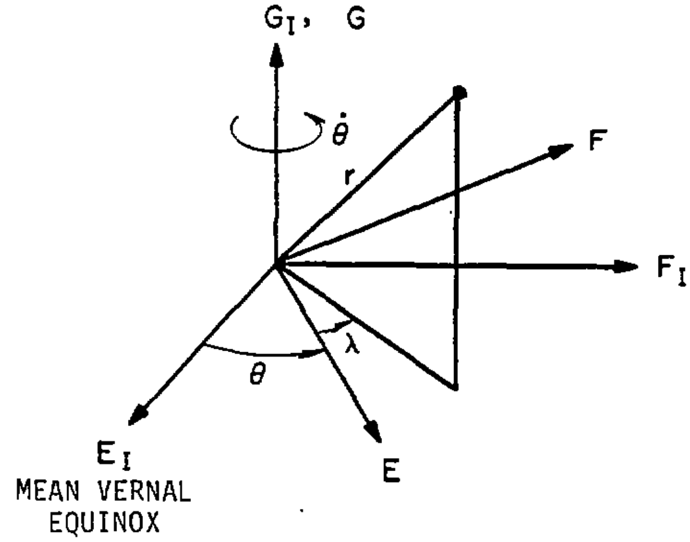


Figure 4-1

The inertial coordinate frame is rotated into the ECEF system as follows:

$$\begin{vmatrix} E \\ F \\ G \end{vmatrix}_R = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} E_I \\ F_I \\ G_I \end{vmatrix}_I \qquad (31)$$

The subscript R on the left-hand side of equation (31) indicates that the EFG (ECEF) coordinate system rotates relative to the inertial (I) system.

The reverse transformation is

$$\begin{vmatrix} E_I \\ F_I \\ G_I \end{vmatrix}_I = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} E \\ F \\ G \end{vmatrix}_R \qquad (32)$$

The transformation of inertial velocities to ECEF velocities is

$$\begin{vmatrix} \dot{E} \\ \dot{F} \\ \dot{G} \end{vmatrix}_R = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \dot{E}_I + \dot{\theta}F_I \\ \dot{F}_I - \dot{\theta}E_I \\ \dot{G}_I \end{vmatrix}_I \qquad (33)$$

The reverse transformation is

$$\begin{vmatrix} \dot{E}_I \\ \dot{F}_I \\ \dot{G}_I \end{vmatrix}_I = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \dot{E} - \dot{\theta}E \\ \dot{F} + \dot{\theta}E \\ \dot{G} \end{vmatrix}_R \tag{34}$$

The transformation of inertial accelerations to ECEF accelerations is

$$\begin{vmatrix} \ddot{E} \\ \ddot{F} \\ \ddot{G} \end{vmatrix}_R = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \ddot{E}_I + 2\dot{\theta}\,\dot{F}_I - \dot{\theta}^2 E_I \\ \ddot{F}_I - 2\dot{\theta}\,\dot{E}_I - \dot{\theta}^2 F_I \\ \ddot{G}_I \end{vmatrix}_I \tag{35}$$

The reverse transformation is

$$\begin{vmatrix} \ddot{E}_I \\ \ddot{F}_I \\ \ddot{G}_I \end{vmatrix}_I = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \ddot{E} - 2\dot{\theta}\,\dot{F} - \dot{\theta}^2 E \\ \ddot{F} + 2\dot{\theta}\,\dot{E} - \dot{\theta}^2 F \\ \ddot{G} \end{vmatrix}_R \tag{36}$$

The matrix in equation (31) transforms the position data of the true of date inertial frame to the true of date ECEF frame by rotating through the hour angle between the true vernal equinox and the Greenwich Meridian. This hour angle is also known as Greenwich Apparent Sidereal Time (θ). The angle θ can be computed from

$$\theta = GAST_o + \omega_e\,(t_i - t_o h_{UT}) \tag{37}$$

where

$GAST_o$ is the Greenwich Apparent Sidereal Time in radians at 0^h Universal Time ($t_o h_{UT}$) on the Julian date of the given epoch,

t_i is the time of interest, and

ω_e is the rotational rate of the Earth measured relative to the instantaneously true vernal equinox:

ω_e = 0.7292115855 x 10^{-4} radians per second

ω_e = 6.300388099 radians per day.

These parameters are illustrated in figure 4-2.

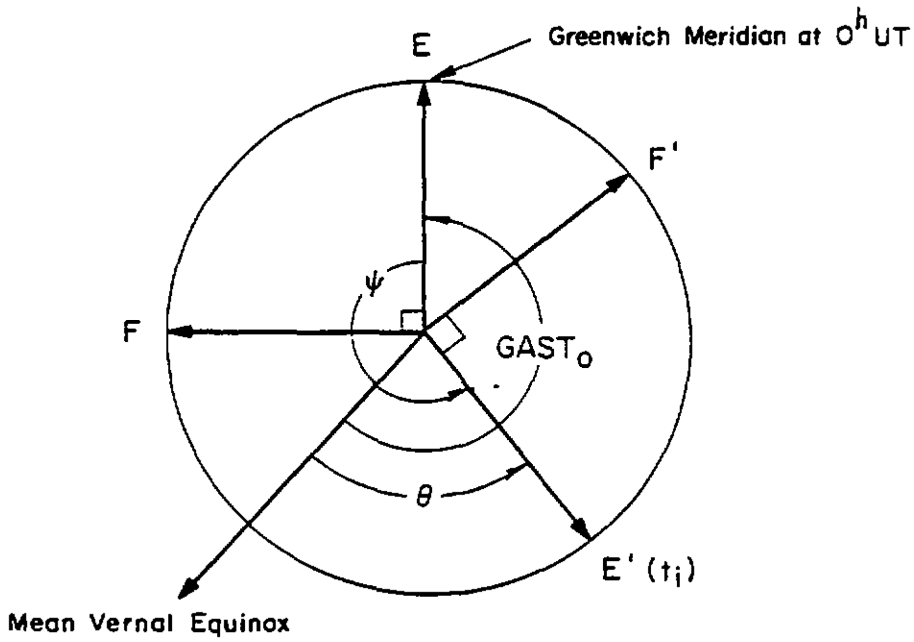


Figure 4-2

The Greenwich Apparent Sidereal Time at 0^h UT, expressed as an angle, represents the angle between the vernal equinox and the Greenwich Meridian. Assuming that the time of interest (t_i) is after midnight, the Greenwich Meridian rotates through the angle ψ to E'. The angle θ represents the Greenwich Apparent Sidereal Time at the time t_i. The Greenwich Mean Sidereal Time in hours at 0^h UT on the Julian date epoch day (JDO_E) is

$$GMST_o \text{ (hours)} = 6.67170278 + 0.0657098232 (JDO_E - 2433282.5) \quad (38)$$

$$\text{MOD } (GMST_o, 24)$$

The mean longitude of the ascending node of the Moon's orbit measured in degrees at 0^h UT is

$$\Omega(\text{Degrees}) = 372.1133 - 0.0529539\ (JDO_E - 2433282.5) \qquad (39)$$

$$\Omega(\text{radians}) = \Omega(\text{Degrees})\ \frac{\pi}{180}$$

The equation of the equinoxes in hours is

$$E_o\ (\text{hours}) = -0.00029 \sin \Omega \qquad (40)$$

The Greenwich Apparent Sidereal Time in hours at 0^h UT is

$$GAST_o\ (\text{hours}) = GMST_o + E_o \qquad (41)$$

$GAST_o$ in radians is

$$GAST_o\ (\text{radians}) = GAST_o\ (\text{hours})\ \frac{2\pi}{24} \qquad (42)$$

The angle θ required for transforming inertial coordinates to ECEF is

$$\theta(\text{radians}) = GAST_o + 6.300388099\ (t_i - t_o h_{UT}) \qquad (43)$$

$$\text{Mod}\ (\theta,\ 2\pi)$$

where

$$t_i - t_o h_{UT} = (JD - 2.4 \times 10^6) - (JDO_E - 2.4 \times 10^6)$$

and JDO_E is the Julian date (JD) at 0^hUT on the day of the epoch (JD is the Julian date of the time of interest). The Julian date is a continuous count of the days and fractions of days from 1 January 4713 B.C., Greenwich Mean Noon (=12^hUT). The JD for t_i may be computed as follows:

$$JD\ (t_i) = 367K - \left\langle \frac{7}{4}\ (K + \langle (M + 9)/12 \rangle) \right\rangle \tag{44}$$

$$+ \left\langle \frac{275M}{9} \right\rangle + I + 1721013.5$$

$$+ UT/24 - 0.5\ \text{sign}\ (100K + M - 190002.5)$$

$$+ 0.5$$

where

K is the year ($1801 \leq K \leq 2099$),

M is the month ($1 \leq M \leq 12$),

I is the day of the month ($1 \leq I \leq 31$),

UT is the Universal Time in hours,

$\langle\ \rangle$ is the integer function, and

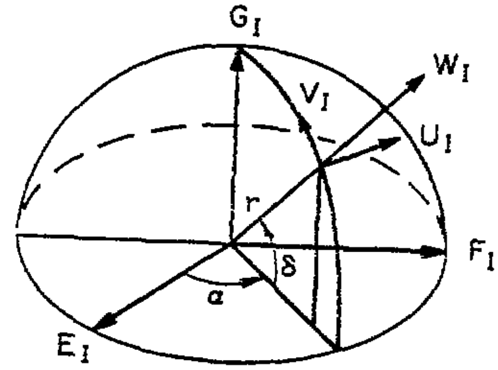
sign is the sign function.

For example, 1978, January 1, 0^h UT = 2443509.5 JD, and

1877, August 11, $7^h\ 30^m$ UT = 2406842.8125 JD

5. Spherical Inertial Coordinate System (r, α, δ, v_I, A_I, γ_I, β_I)

A spherical system is assumed with the origin the same as the ECI (E_I, F_I, G_I) coordinate system. This coordinate system is right-handed with E_I positive toward the mean vernal equinox of date, F_I 90° counterclockwise from E_I, and G_I coincident with the Earth's rotational axis and positive toward the North Pole (see figure 5-1).



VERNAL EQUINOX

Figure 5-1

A coordinate system designated U_I, V_I, W_I, is also shown in figure 5-1. This coordinate system is right-handed with U_I pointing east, V_I pointing north, and W_I pointing along the projection of the geocentric radius r. The $(U\text{-}V)_I$ plane is normal to the geocentric radius. The following definitions apply to the parameters used in conjunction with the Spherical Inertial Coordinate System:

r is the geocentric range from the center of the Earth to the point (origin of the $(UVW)_I$ coordinate system).

δ is the declination, positive north. The declination is the angle between the equatorial plane and the range vector.

α is the right ascension measured positive east from the vernal equinox of date.

A_I is the azimuth of the velocity vector projected on the $(U\text{-}V)_I$ plane.

γ_I is the elevation of the velocity vector above the $(U\text{-}V)_I$ plane.

β_I is the angle between the range vector and the velocity vector.

If the position vector components of a point in space are given in the ECI system, then r, α, δ may be found from the following:

$$r = (E_I^2 + F_I^2 + G_I^2)^{1/2} \tag{45}$$

$$\alpha = \tan^{-1}\left(\frac{F_I}{E_I}\right); \quad \text{(Double Arctan)} \; (\text{If } \alpha < 0, \text{ then } \alpha = \alpha + 2\pi) \tag{46}$$

$$\delta = \tan^{-1}\left(\frac{G_I}{(E_I^2 + F_I^2)^{1/2}}\right) \tag{47}$$

where

E_I, F_I, and G_I are geocentric inertial coordinates of the point.

When the coordinates of the point in space are given in polar coordinates (r, α, δ), the geocentric coordinates are

$$E_I = r \cos \delta \cos \alpha \tag{48}$$

$$F_I = r \cos \delta \sin \alpha$$

$$G_I = r \sin \delta$$

If the geocentric inertial velocities are known, they can be transformed from the ECI, $(EFG)_I$ system to the $(UVW)_I$ coordinate system with the following rotations:

$$\theta_1 = \alpha$$

$$\theta_2 = 90°$$

$$\theta_3 = (90-\delta)$$

Substituting the above angles into equation (22) gives

$$(R_3R_2R_1)_I = \begin{vmatrix} -\sin \alpha & \cos \alpha & 0 \\ -\sin \delta \cos \alpha & -\sin \delta \sin \alpha & \cos \delta \\ \cos \delta \cos \alpha & \cos \delta \sin \alpha & \sin \delta \end{vmatrix} \tag{49}$$

The transformation is

$$\begin{vmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{vmatrix}_I = (R_3R_2R_1)_I \begin{vmatrix} \dot{E}_I \\ \dot{F}_I \\ \dot{G}_I \end{vmatrix} \tag{50}$$

or

$$\dot{U}_I = -\dot{E}_I \sin \alpha + \dot{F}_I \cos \alpha$$

$$\dot{V}_I = -\dot{E}_I \sin \delta \cos \alpha - \dot{F}_I \sin \delta \sin \alpha + \dot{G}_I \cos \delta \qquad (51)$$

$$\dot{W}_I = \dot{E}_I \cos \delta \cos \alpha + \dot{F}_I \cos \delta \sin \alpha + \dot{G}_I \sin \delta$$

The transformed velocities may be used to find the total velocity (v_I) as well as A_I, γ_I, β_I from the following equations (see figure 5-2):

$$v_I = (\dot{U}_I^2 + \dot{V}_I^2 + \dot{W}_I^2)^{1/2} \qquad (52)$$

$$A_I = \tan^{-1}\left(\frac{\dot{U}_I}{\dot{V}_I}\right) \quad \text{(Double Arctan)} \quad (\text{If } A_I < 0, \text{ then } A_I = A_I + 2\pi) \qquad (53)$$

$$\gamma_I = \tan^{-1}\left(\frac{\dot{W}_I}{(\dot{U}_I^2 + \dot{V}_I^2)^{1/2}}\right) \qquad (54)$$

$$\beta_I = 90° + \gamma_I \qquad (55)$$

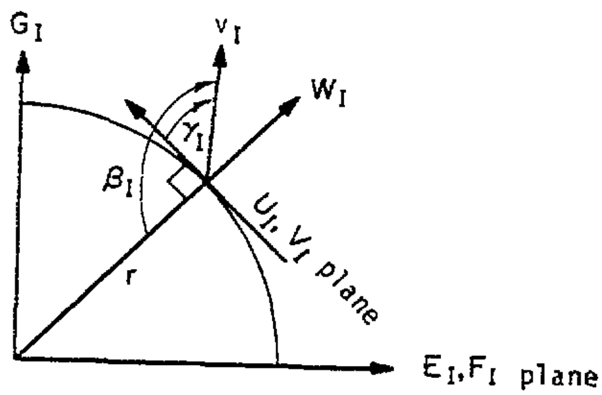


Figure 5-2

If, on the other hand, v_I, A_I, and β_I are known, then

$$\dot{U}_I = v_I \sin \beta_I \sin A_I$$
$$\dot{V}_I = v_I \sin \beta_I \cos A_I \qquad (56)$$
$$\dot{W}_I = -v_I \cos \beta_I$$

or

$$\dot{U}_I = v_I \cos \gamma_I \sin A_I$$
$$\dot{V}_I = v_I \cos \gamma_I \cos A_I \qquad (57)$$
$$\dot{W}_I = v_I \sin \gamma_I$$

In this case, the geocentric inertial velocities may be obtained with the following matrix transformations:

$$\begin{vmatrix} \dot{E}_I \\ \dot{F}_I \\ \dot{G}_I \end{vmatrix} = (R_3 R_2 R_1)_I^T \begin{vmatrix} \dot{U}_I \\ \dot{V}_I \\ \dot{W}_I \end{vmatrix} \qquad (58)$$

6. <u>Satellite Orbital Elements System (Ω, ω, i, a, e, M)</u>

At any instant, the position and motion of an artificial satellite, in earth-centered inertially fixed coordinates, can be described by the rectangular components of position (E_I, F_I, G_I) and velocity ($\dot{E}_I$, $\dot{F}_I$, $\dot{G}_I$). The satellite's position at a given time can also be described by six elements of the Keplerian ellipse (Ω, ω, i, a, e, M). Three of these elements specify the spacial position of the orbital plane, two give the size and shape of the orbit, and the sixth relates orbital position to time. These elements are defined as follows:

Ω is the right ascension of the ascending node.

ω is the argument of perigee.

i is the inclination of the plane of the orbit with respect to the equatorial plane.

a is the semi-major axis of the elliptical orbit.

e is the eccentricity of the elliptical orbit.

M is the mean anomaly.

ε is the eccentric anomaly.

E_I, F_I, G_I, are the inertial geocentric coordinates of the satellite.

$\dot{E}_I$, $\dot{F}_I$, $\dot{G}_I$, are the inertial geocentric velocities components of the satellite.

Figure 6-1 indicates the coordinate system discussed in this section. Figure 6-2 shows the relationship of the position of a satellite in its elliptical orbit and its projection onto an auxiliary circle (dotted line). The symbols X_ω and Y_ω are orbital plane coordinates, ε is the eccentric anomaly that is measured in the orbital plane from perigee to S', f is the true anomaly (not to be confused with flattening), and ρ is the radius from the center of the Earth to the satellite.

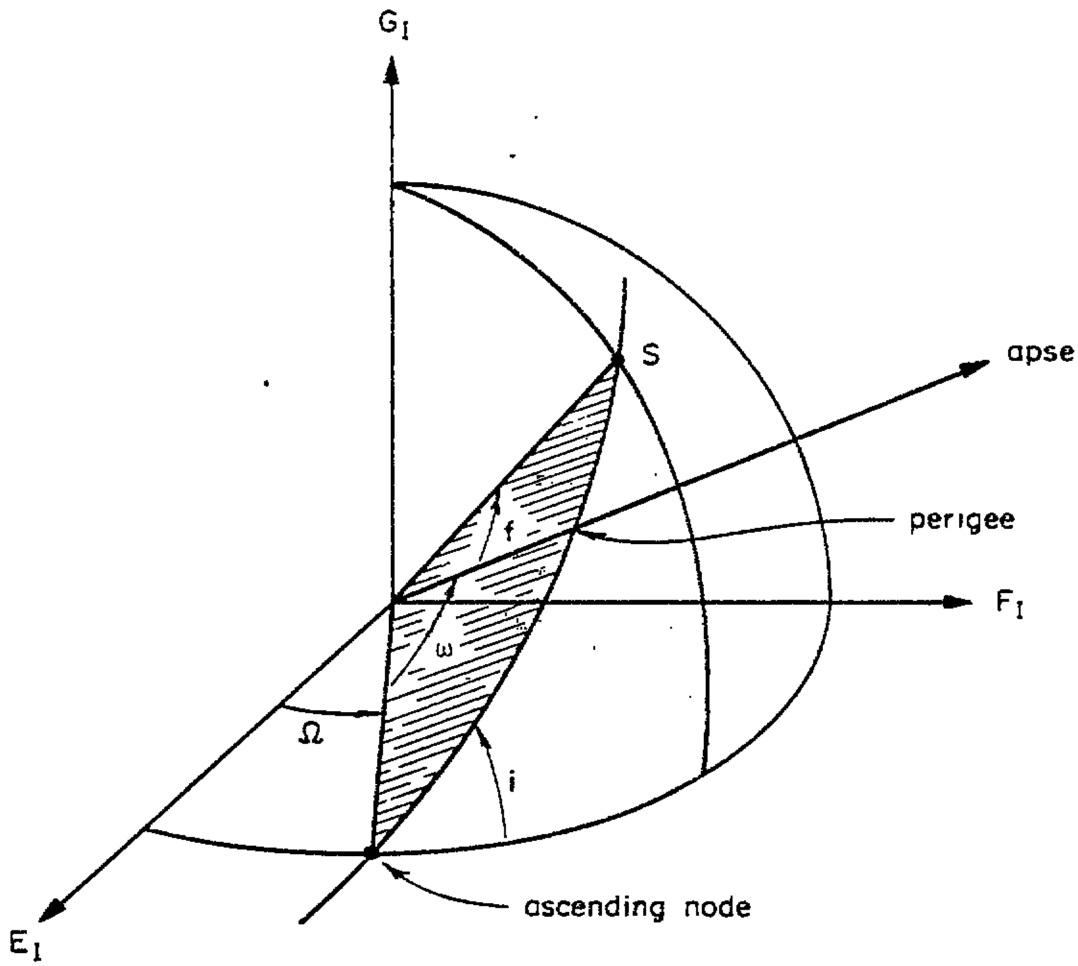


Figure 6-1

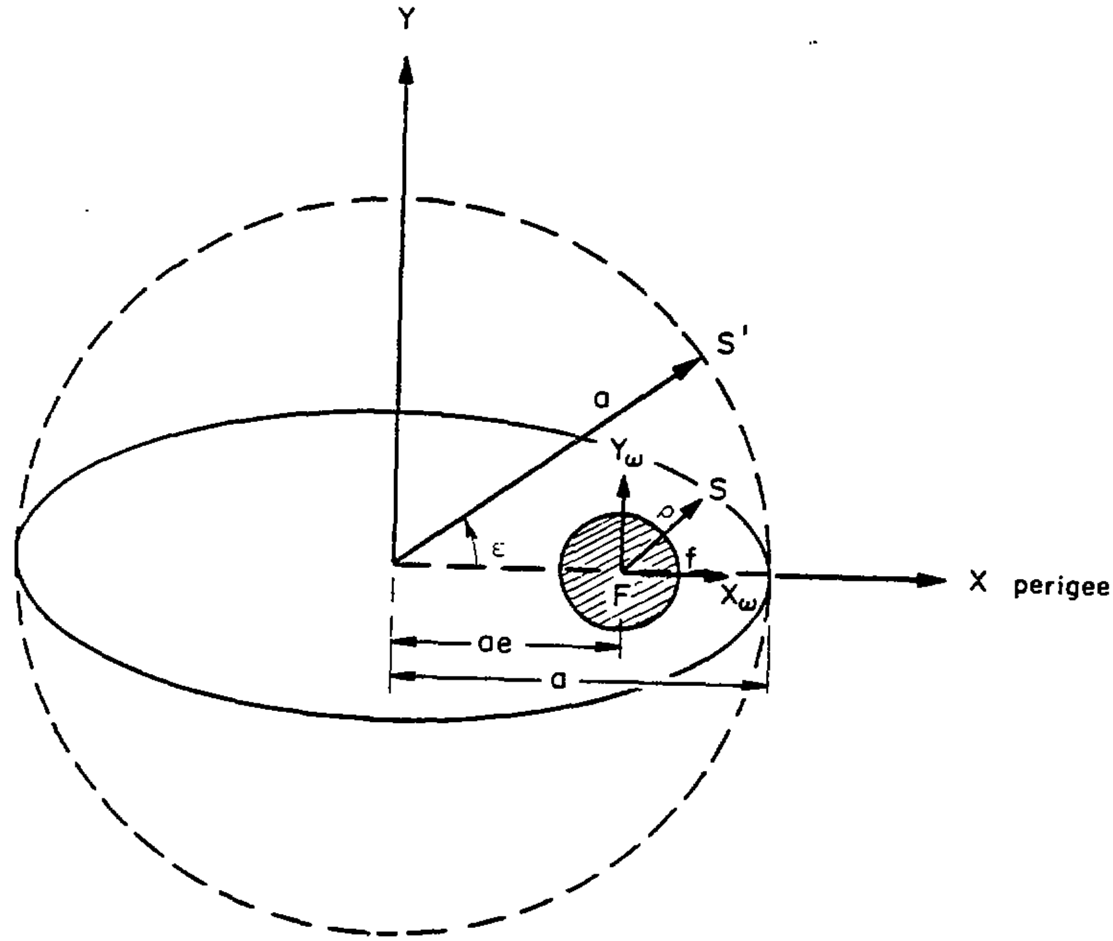


Figure 6-2

The coordinates of S with respect to the origin of the $(EFG)_I$ coordinate system are

$$X_\omega = \rho \cos f$$
$$Y_\omega = \rho \sin f \qquad (59)$$
$$Z_\omega = 0$$

The orbital coordinates of S expressed as a function of the eccentric anomaly are

$$X_\omega = a\ (\cos \varepsilon - e)$$
$$Y_\omega = a\ (1 - e^2)^{\frac{1}{2}} \sin \varepsilon \qquad (60)$$
$$Z_\omega = 0$$

The orbital velocities are

$$\dot{X}_\omega = \left\{\frac{-\sin \varepsilon}{(1-e \cos \varepsilon)}\right\} \left(\frac{\mu}{a}\right)^{1/2}$$

$$\dot{Y}_\omega = \left\{\frac{(1-e^2)^{1/2} \cos \varepsilon}{(1-e \cos \varepsilon)}\right\} \left(\frac{\mu}{a}\right)^{1/2} \quad (61)$$

$$\dot{Z}_\omega = 0$$

where

μ is the gravitational constant and

ε is the eccentric anomaly, which must be obtained by a solution of

$$M = \varepsilon - e \sin \varepsilon \quad (62)$$

The equation is usually solved by iteration. The first approximation of ε is

$$\varepsilon_1 = M + e \sin M + \frac{1}{2} e^2 \sin 2M$$

Then

$$M_1 = \varepsilon_1 - e \sin \varepsilon_1 \quad (63)$$

$$\Delta M_1 = M - M_1$$

$$\Delta\varepsilon_1 = \frac{\Delta M_1}{(1-e \cos \varepsilon_1)}$$

Add the $\Delta\varepsilon_1$ to ε_1 to give ε_2. Repeat until $M_n = M$.

The transformation from orbital coordinates to inertial geocentric coordinates is

$$\begin{vmatrix} E_I \\ F_I \\ G_I \end{vmatrix} = \begin{vmatrix} \cos \Omega & -\sin \Omega & 0 \\ \sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{vmatrix} \begin{vmatrix} \cos \omega & -\sin \omega & 0 \\ \sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} X_\omega \\ Y_\omega \\ Z_\omega \end{vmatrix}$$

(64)

$$\begin{vmatrix} \dot{E}_I \\ \dot{F}_I \\ \dot{G}_I \end{vmatrix} = \begin{vmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{vmatrix} \begin{vmatrix} \dot{X}_\omega \\ \dot{Y}_\omega \\ \dot{Z}_\omega \end{vmatrix} \quad (65)$$

where

P_{11} = cos ω cos Ω -sin ω sin Ω cos i,

P_{12} = -sin ω cos Ω -cos ω sin Ω cos i,

P_{13} = sin Ω sin i,

P_{21} = cos ω sin Ω + sin ω cos Ω cos i,

P_{22} = -sin ω sin Ω + cos ω cos Ω cos i,

P_{23} = -cos Ω sin i,

P_{31} = sin Ω sin i,

P_{32} = cos ω sin i, and

P_{33} = cos i.

Calling the above matrix the P matrix, the transformation from orbital velocities to inertial geocentric velocities is

$$\begin{vmatrix} \dot{E}_I \\ \dot{F}_I \\ \dot{G}_I \end{vmatrix} = P \begin{vmatrix} \dot{X}_\omega \\ \dot{Y}_\omega \\ \dot{Z}_\omega \end{vmatrix} \quad (66)$$

Assuming that the position and velocity vectors exist in the ECI system, the orbital elements (Ω, ω, i, a, e, M) can be determined. The semi-major axis of the elliptical orbit is

$$a = \frac{\mu r}{2\mu - r\, v_I^{\,2}} \quad (67)$$

where

$$r = (E_I^2 + F_I^2 + G_I^2)^{1/2} \quad \text{and,} \tag{68}$$

$$v_I = (\dot{E}_I^2 + \dot{F}_I^2 + \dot{G}_I^2)^{1/2} \tag{69}$$

Next, calculate the angular momentum vector $\vec{L}$:

$$\vec{L} = \vec{r} \times \vec{v}_I \tag{70}$$

along with

$$\begin{aligned} \ell_x &= F_I\dot{G}_I - G_I\dot{F}_I \\ \ell_y &= G_I\dot{E}_I - E_I\dot{G}_I \\ \ell_z &= E_I\dot{F}_I - F_I\dot{E}_I \end{aligned} \tag{71}$$

The parameter $\vec{\ell}$ defines a unit vector in the direction of $\vec{L}$ with components

$$\ell_x/L, \ \ell_y/L \text{ and } \ell_z/L, \tag{72}$$

where

$$L = (\ell_x^2 + \ell_y^2 + \ell_z^2)^{1/2} .$$

Rotating the position coordinates through Ω and i produces

$$\begin{vmatrix} E' \\ F' \\ G' \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{vmatrix} \begin{vmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} E_I \\ F_I \\ G_I \end{vmatrix} \tag{73}$$

$$\begin{vmatrix} E' \\ F' \\ G' \end{vmatrix} = \begin{vmatrix} \cos\Omega & \sin\Omega & 0 \\ -\cos i \sin\Omega & \cos i \cos\Omega & \sin i \\ \sin i \sin\Omega & -\sin i \cos\Omega & \cos i \end{vmatrix} \begin{vmatrix} E_I \\ F_I \\ G_I \end{vmatrix} \tag{74}$$

Solving for G' gives

$$G' = (\sin i \sin \Omega)\, E_I + (-\sin i \cos \Omega)\, F_I + (\cos i)\, G_I \tag{75}$$

Equating the like expressions for ℓ' and G' gives

$$\begin{aligned} \ell_x/L &= \sin i \sin \Omega \\ \ell_y/L &= -\sin i \cos \Omega \\ \ell_z/L &= \cos i \end{aligned} \tag{76}$$

Solving for Ω from the above expressions gives

$$\Omega = \tan^{-1} \left[\frac{-\ell_x}{\ell_y}\right] \quad \begin{matrix}\text{(Double Arctan)} \\ \text{(If } \Omega < 0\text{, then } \Omega = \Omega + 2\pi)\end{matrix} \tag{77}$$

$$i = \tan^{-1} \left(\frac{\left(\ell_x^2 + \ell_y^2\right)^{\frac{1}{2}}}{\ell_z}\right). \tag{78}$$

The eccentricity (e) can be calculated from standard formulas involving the semi-major axis and flattening or the semi-major and semi-minor axes of the ellipse. The eccentric anomaly (ε) then follows as

$$e \cos \varepsilon = \left[1 - \frac{r}{a}\right] \tag{79}$$

$$e \sin \varepsilon = (E_I \dot{E}_I + F_I \dot{F}_I + G_I \dot{G}_I)/(\mu a)^{\frac{1}{2}} \tag{80}$$

$$\varepsilon = \tan^{-1} \left(\frac{r\dot{r}(\mu a)^{\frac{1}{2}}}{\mu(a-r)}\right) = \tan^{-1} \left[\frac{e \sin \varepsilon}{e \cos \varepsilon}\right] \tag{81}$$

The term $\dot{r}$ is the magnitude of the radial component of velocity. Often this quantity is not readily available; however, the radius vector ($\vec{r}$) and the total inertial velocity vector ($\vec{v}_I$) are generally available. When this is the case, the term $r\,\dot{r}$ can be replaced by $\vec{r}\cdot\vec{v}_I$ (dot product) in equation (81). In any event, the term $\dot{r}$ should not be confused with v_I. Finally, compute the mean anomaly as

$$M = \varepsilon - e \sin \varepsilon. \tag{82}$$

To determine the argument of perigee (ω) find the angle between the line of nodes and the satellite position measured in the orbital plane θ.

From equation (74)

$$E' = E_I \cos \Omega + F_I \sin \Omega \tag{83}$$

$$F' = -E_I \cos i \sin \Omega + F_I \cos i \cos \Omega + G_I \sin i$$

$$\theta = \tan^{-1}\left(\frac{F'}{E'}\right) \begin{array}{l}\text{(Double Arctan)} \\ \text{(If } \theta < 0\text{, then } \theta = \theta + 2\pi\text{)}\end{array} \tag{84}$$

Now find the true anomaly (f), the angle between perigee and the satellite position:

$$f = 2 \tan^{-1}\left(\left(\frac{1 + e}{1 - e}\right)^{1/2} \frac{\sin \frac{\varepsilon}{2}}{\cos \frac{\varepsilon}{2}}\right) \quad \text{(Double Arctan)} \tag{85}$$

or

$$f = \tan^{-1}\left(\frac{L\left(E_I\dot{E}_I + F_I\dot{F}_I + G_I\dot{G}_I\right)}{L^2 - \mu r}\right) \begin{array}{l}\text{(Double Arctan)} \\ \text{(If } f < 0\text{, then } f = f + 2\pi\text{)}\end{array} \tag{86}$$

Now finally

$$\omega = \theta - f \quad (\text{If } \omega < 0\text{, then } \omega = \omega + 2\pi) \tag{87}$$

It is advisable to exercise due care in determining the quadrants of the angular quantities ω, θ, and f. A way of making this determination is to compute both the sine and cosine value for a given angle.

7. Geodetic Coordinate System (λ, ϕ, H)

The geodetic coordinate system is related to the geocentric coordinate system as shown in figure 7-1. The geodetic latitude (ϕ) is the angle between the geodetic Equator and the geodetic line that is normal to the surface of the ellipsoid at any point. The geodetic latitude is positive in the northern half of the hemisphere. The geodetic longitude (λ) of a point is measured positive to the east of the Greenwich Meridian. The angle is measured in the E-F plane between the zero meridian plane and the geodetic meridian plane of the point. Both planes contain the minor axis of the reference ellipsoid.

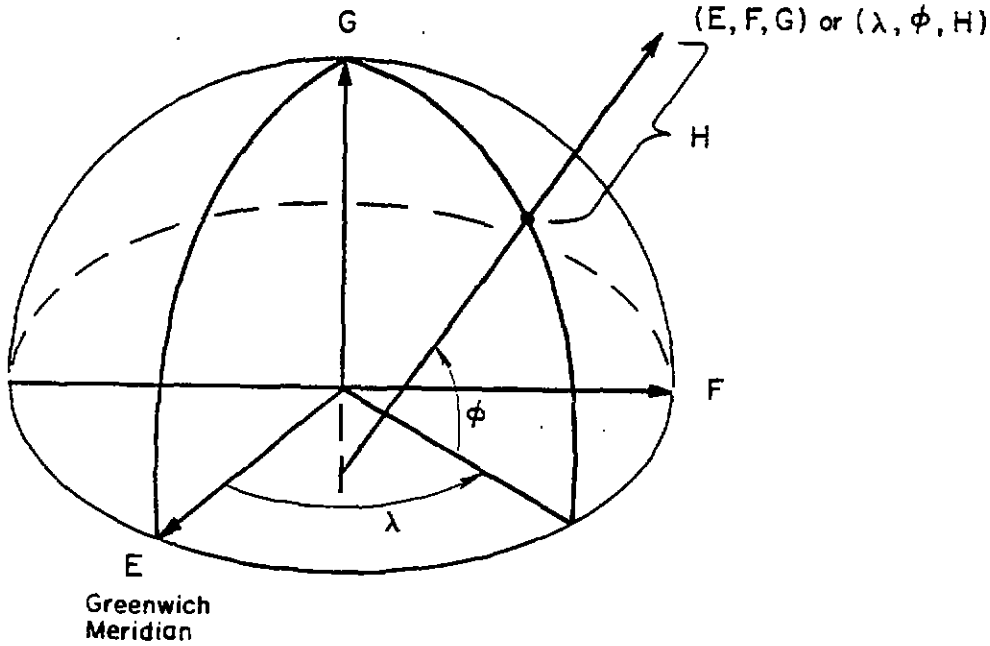


Figure 7-1

The geocentric coordinates of a point in space may be obtained from the geodetic coordinates as follows:

$$E = (N + H) \cos \phi \cos \lambda$$

$$F = (N + H) \cos \phi \sin \lambda \tag{88}$$

$$G = [N (1-e^2) + H] \sin \phi,$$

where N is the radius of curvature in the prime vertical

$$N = \frac{a}{(1-e^2 \sin \phi)^{\frac{1}{2}}} \tag{89}$$

In this relation, a and e are the semi-major axis and eccentricity, respectively, for the reference ellipsoid.

If the geocentric coordinates are known, the reverse information is

$$\lambda = \tan^{-1}\left(\frac{F}{E}\right) \quad \begin{array}{l}\text{(Double Arctan)} \\ \text{(If } \lambda < 0\text{, then } \lambda = \lambda + 2\pi\text{)}\end{array} \tag{90}$$

Latitude is obtained by Newton's iterative method as follows:

$$\phi_{j+1} = \phi_j - f_j \left(\frac{\partial f}{\partial \phi}\right)_j^{-1} \tag{91}$$

where

$$f_j = \frac{ae^2}{(1-e^2 \sin^2 \phi_j)^{1/2}} - \frac{(E^2 + F^2)^{1/2}}{\cos \phi_j} + \frac{G}{\sin \phi_j} \quad \text{and} \tag{92}$$

$$\left(\frac{\partial f}{\partial \phi}\right)_j = \frac{e^4 a \sin \phi_j \cos \phi_j}{(1-e^2 \sin^2 \phi_j)^{3/2}} - \frac{(E^2 + F^2)^{1/2} \sin \phi_j}{\cos^2 \phi_j} - \frac{G \cos \phi_j}{\sin^2 \phi_j} . \tag{93}$$

For $j = 1$

$$\phi_j = \tan^{-1}\left(\frac{G}{(1-e^2)\, D}\right) \tag{94}$$

where $D = (E^2 + F^2)^{1/2}$.

The solution is complete when $-f_j \left(\frac{\partial f}{\partial \phi}\right)_j^{-1}$ is $< 10^{-12}$ radians. Then

$$H = \pm \left[(D-D_s)^2 + (G - G_s)^2\right]^{1/2} \tag{95}$$

where

$$D_s = N \cos \phi \quad \text{and} \tag{96}$$

$$G_s = N\,(1-e^2) \sin \phi .$$

To resolve the ambiguity in sign, compute

$$B = D_s\,(D-D_s) + G_s\,(G-G_s) \tag{97}$$

The sign in the computed value of B is then assigned to H. As an alternative, H can be computed as

$$H = [D^2 + (G + e^2 N \sin \phi)^2]^{\frac{1}{2}} - [D_S^2 + (N \sin \phi)^2]^{\frac{1}{2}} \qquad (98)$$

This expression yields the sign of H directly as part of the computation.

8. Summary of Trajectory Coordinate Systems

Figure 8-1 depicts the relationships of the trajectory coordinate systems defined in section VII.

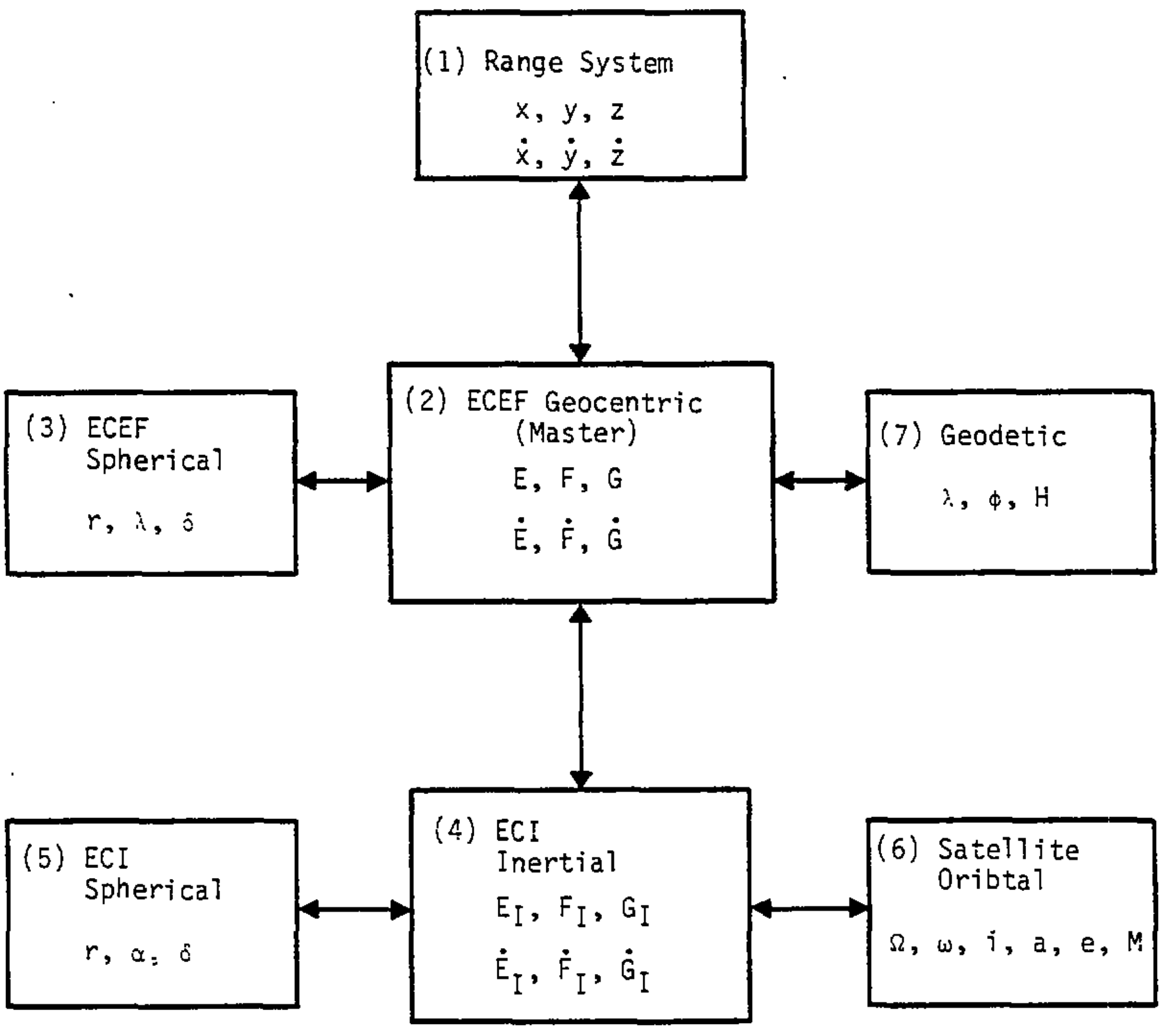


Figure 8-1

VIII. RCC RECOMMENDATIONS AND PROCEDURES

The Range Commanders Council adopts and recommends for interrange use the Geodetic Datum and Global Coordinate System described below.

a. For the exchange of data among ranges on missions where interrange action is required, employ the Department of Defense World Geodetic System 1984, with the following ellipsoid definitions:

WGS 84 ELLIPSOID

PARAMETERS	NOTATION	MAGNITUDE	STANDARD ERROR (68.27%)
Gravitational[1]* Constant	GM	398600.5 km^3/s^2	± 0.06
Second Degree* Zonal	$\overline{C}_{2,0}$	$-484.16685 \times 10^{-6}$	$\pm 1.30 \times 10^{-9}$
Angular Velocity[2]*	ω_e	0.7292115×10^{-4} rad/s	$\pm 0.1500 \times 10^{-11}$
Angular Velocity[3]*	ω_e'	$0.72921158553 \times 10^{-4}$ $+4.3(10^{-15}\ T_\mu)$ rad/s	$\pm 0.1500 \times 10^{-11}$
Gravitational[4] Constant	GM′	398600.15 km^3/s^2	± 0.6
Semi-major Axis*	a	6378137 m	± 2 m
Flattening (Ellipticity)	f	1/298.257223563 (0.00335281066474)	-----
First eccentricity	e	0.0818191908426	
	e^2	0.00669437999013	

*Defining parameters of WGS 84 Ellipsoid (see reference 1).

[1]Contains the mass of the Earth's atmosphere (for use with satellite and space studies).

[2]IAG adopted value for the true angular velocity of the earth.

[3]Relative to the instantaneous true equinox; T_μ = Julian Centuries from Epoch J2000.

[4]Excluding atmosphere (for use with geodetic computations involving the normal potential).

b. Transformation constants for shifting from three regional datums to the WGS 84 Datum are given below. The unclassified regional datum shifts listed are taken from reference 1.

TRANSFORMATION	E(m)	F(m)	G(m)	a(m)	f x 10
North American* Datum (NAD) 27 Area to WGS-84	-8	160	176	-69.4	-0.37264639
European Datum (ED) 1950 (International) Area to WGS-84	-87	-98	-121	-251	-0.14192702
Tokyo Datum (TD) (Bessel) Area to WGS-84	-128	481	664	739.845	0.10037483

*Mean Value (CONUS).

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